measnet

Network of European Measuring Institutes

CUP ANEMOMETER CALIBRATION PROCEDURE

Version 1
September 1997

(Text updated 25/11/2008)

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1 FOREWORD

MEASNET is a network of measurement institutes which have been established to harmonise wind energy related measurement procedures within the European Union. The institutes of MEASNET are all actively performing wind energy related measurements. Each institute has to document the skills and quality of measurements, to apply agreed "MEASNET measurement procedures" and to participate as required in mutual evaluation exercises.

2 INTRODUCTION

In the following, the requirements and the procedure to be followed for **calibration of cup anemometers** in a wind tunnel are described. The requirements are based on the "Recommendations on the use and calibration of cup anemometers" [1], carried out in the framework of the European Wind Turbine Standards Joule project. The results of this project proved the necessity of introducing a transfer standard as well as a uniform interpretation of the existing recommendations in the anemometer calibration procedure in order to allow for mutual recognition and interchangeability of the results.

The **intention** is to give an adequate framework for wind tunnel calibrations of cup anemometers in order to meet the MEASNET requirements [2] for power performance measurements and other wind energy related applications. According to the MEASNET procedure it is essential to obtain a calibration with quantified, traceable accuracy for an anemometer prior to its use.

The calibration procedure that is followed features:

- the use of a common reference as a mean for verification of the results obtained at different wind tunnels;
- the application of the ISO Guide for the Expression of Uncertainty as the basis of uncertainty estimations;
- traceability of the instrumentation used.

3 GENERAL REQUIREMENTS

The general requirements for anemometer calibration are summarised as follows:

- all transducers and measuring equipment shall have **traceable calibrations**. Calibration certificates and reports shall obtain **all** relevant traceability information. All reference standards used during the calibration of the anemometer shall be stated within the test report of the calibration campaign;
- the Pitot tubes used shall comply with ISO 3966 [5];
- prior to every calibration round the integrity of the experimental set-up shall be verified by means of comparative calibration of a "**reference anemometer**" of the institute;
- **flow quality measurement** shall be carried out;
- the **repeatability** of the calibration shall be verified;

- anemometer calibration shall be supported by a thorough assessment of **calibration uncertainty**, carried out in accordance with ISO guidelines [3].

4 REQUIREMENTS OF THE WIND TUNNEL

The wind tunnel shall be well equipped and carefully prepared to carry out accurate anemometer calibrations.

The presence of the anemometer shall not affect substantially the flow field in the wind tunnel. During measurements the anemometer will to some extent be influenced by wind tunnel blockage or boundary effects. The **blockage ratio** - defined as the ratio of the anemometer frontal area (including its mounting system) to the total test section area - shall not exceed 0.1 for open test section and 0.05 for closed test section.

The flow across the anemometer frontage shall be uniform. The **flow uniformity** shall be assessed prior to the anemometer's calibration. Flow uniformity can be estimated using velocity sensing devices, i.e. Pitot tubes, hot wires or Laser Doppler velocimetry and measuring flow profiles in longitudinal, transversal and vertical direction. The flow shall be uniform to 0.2% across the area covered by the anemometer. These investigations shall be carried out for the wind tunnel once and additionally after each modification of the wind tunnel aerodynamics.

Cup anemometers are very sensitive to horizontal wind gradients. Different horizontal wind gradients can be seen depending on pollution of nets and smoothing devices. Therefore it is useful to check the horizontal wind gradient by using two identical Pitot tubes. They shall be placed at the exact position where the anemometer will be placed with their heads spanning approximately the area covered by the cup anemometers rotating cups. A set of measurements shall be made and the linear regression between the dynamic pressures measured by the two Pitot tubes shall be calculated. The flow shall be uniform to 0.2%. This experiment shall be carried out for each calibration campaign.

The axial **turbulence intensity** at the anemometer's position shall be below 2%.

The wind tunnel **calibration factor** which gives the relation between the conditions at the reference measurement position and those at the anemometer position shall be appraised using Pitot tubes.

The facility shall, as a matter of course, undergo a detailed examination of the **repeatability** of anemometer calibrations. This shall include at least 5 calibrations of the reference anemometer (over various atmospheric conditions) with a target maximum difference between calibrations of less than 0.5 % at 10 m/s wind speed. The average value of the calibrations shall not deviate by more than 1 % at 10 m/s wind speed in a common reference wind tunnel (reproducibility).

This process shall be repeated after any modification or recalibration of the facility.

If different **operators** are using the facility, then it must be demonstrated that there is no significant bias in the calibrations which the different operators produce.

5 INSTRUMENTATION (CALIBRATION SET-UP REQUIREMENTS)

Dedicated external **signal conditioning equipment** such as frequency to voltage converters etc. shall be calibrated in isolation from the anemometer, so allowing the anemometer's calibration to be derived and reported in isolation from the signal conditioning equipment.

The resolution of the **data acquisition system** used shall be at least of 50 bits at 10 m/s per m/s. Care shall also be exercised in the case of an analogue voltage instrument, to ensure that the signal is adequately buffered to prevent its attenuation by low impedance logging equipment. Such effects are easy to overlook since 'believable' signals are still registered.

During calibration the anemometer shall be mounted on top of a tube in order to minimise **flow distortion**. This tube shall be of the same dimensions as the one on which the anemometer will be mounted in service in the free atmosphere. Mounting arrangements can have dramatic effects on instrument sensitivity, particularly if the ratio of tube diameter to rotor diameter is high.

It is important to ensure that the anemometer is not influenced by the presence of any reference wind speed measurement equipment. Conversely the presence of the anemometer shall not affect the flow in the region of the reference instrument. If **flow distortion** effects are encountered then the Pitot tube shall be repositioned. This effect can be assessed by removing and the reinstating the anemometer and afterwards the reference instrument (be it a Pitot tube or a reference anemometer), and ascertaining whether the output of the remaining instrument changes. To remove uncertainty caused by uncontrolled drift of the tunnel, it is suggested that this procedure is repeated several times.

The **Pitot tubes** shall be positioned at the test section perpendicular to the flow field of the wind tunnel as accurate as possible. The maximum declination allowed is 1°.

The anemometer shall be positioned at the test section perpendicular to the flow field of the wind tunnel as accurate as possible. The maximum deviation allowed is 1^{o}. A number of studies have shown that a cup anemometer's **sensitivity to vertical angle of attack** is generally non-cosine and depends upon the instrument's geometry.

During calibration the anemometer **output signal** shall be examined to ensure that it is not subjected to interference or noise.

6 CALIBRATION PROCEDURE

The anemometer shall run in for about 5 minutes before the calibration procedure begins in order to avoid the effect that large temperature variations may have on the mechanical friction of the anemometer bearings. Calibration shall be performed under both rising and falling wind speed in the range of 4 to 16 m/sec at a **calibration interval** of 1m/s or less. By taking readings both for increasing steps and for decreasing steps, it is possible to identify whether hysteresis effects are present in the measuring equipment. (Note: 1 m/s intervals can also be realised with the allowance for 2 m/s jumps, e.g. 4, 6, 8, 10, 12, 14, 16, 15, 13, 11, 9, 7, 5 m/s).

The **sampling frequency** shall be at least 1 Hz and the sampling interval 30 sec. This time shall be increased when low resolution anemometers are calibrated. It is important to ensure that anemometer and reference wind speed readings span the same period of time. Before collecting data at each wind speed, adequate time shall be allowed for stable flow conditions to become established. This will typically take 1 minute, but will vary from facility to facility. **Stability** can be assumed if two successive 30 second means are within 0.05 m/s of each other.

Air density ρ shall be calculated based on the mean wind tunnel air temperature $\overline{T}$ (^{o} K), humidity $\overline{\phi}$ and barometric pressure B (Pa). A precise formula (standard uncertainty less than 10^{-4} kg/m^3) for the expression of ρ is given in Ref. 7. A more simple, but less precise (standard uncertainty less than 10^{-3} kg/m^3), formula is:

$$\rho = \frac{1}{T}\left(\frac{B}{R_0} - \phi P_w\left(\frac{1}{R_0} - \frac{1}{R_w}\right)\right)$$

where

B = the barometric pressure (Pa)

T = the absolute temperature (^{O}K)

R_0 = the gas constant of dry air (287,05 J/kgK)

R_w = the gas constant of water vapour (461,5 J/kgK)

P_w = the vapour pressure

$$P_w = 0{,}0000205\exp(0{,}0631846 \cdot T)$$

where **vapour pressure** P_W (Pa) depends on mean air temperature.

The **mean flow speed** at anemometer position is calculated from mean differential pressure Δp_{ref} at reference position using equation (2)

$$\bar{v} = k_b \frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{2k_c}{C_h}\frac{\Delta p_{REF,i\,i}}{\rho}} \qquad (2)$$

where

C_h : Pitot tube head coefficient

k_C : wind tunnel calibration factor as previously defined

k_b : Blockage correction factor

n : number of samples within the sampling interval

R_A : individual Gas Constant of dry air = 287.05 [J/(kg K)]

The **Blockage correction factor** for the cases of enclosed wind tunnels should be calculated using Maskells theorem [4]. If no blockage correction factor is calculated, then about 1/4 of the blockage ratio shall be used for the uncertainty calculation for closed wind tunnels and 1/16 for open wind tunnels.

7 DATA ANALYSIS

A **linear regression analysis** shall be carried out on the calibration data for the estimation of the following regression parameters: Offset, slope, regression coefficient, standard uncertainty in the slope and intercept and the covariance of the slope and intercept of the wind speed, as described in Appendix I. The wind speed values shall be regressed upon the anemometer outputs. Although it may seem logical to regress anemometer output on wind speed, it is more convenient to do the reverse. During calibration the anemometer output is normally known to a high degree of accuracy, whereas the wind speed measurement is much less certain.

If the **correlation coefficient** for the data is less than 0.99995 then the calibration shall be repeated. If the coefficient is still insufficiently high, then either the calibration facility is inadequate or the anemometer is excessively non linear and shall not be used unless it is feasible to employ an unlinear calibration curve. If it is intended to use a higher order curve, then first the linearity of all instrumentation shall be checked to ensure the non-linearity is indeed due to the anemometer. If a higher order calibration curve is to be used then the relevant parameters shall be calculated and included within the calibration certificate or report.

8 UNCERTAINTY ANALYSIS

It is vital to identify the uncertainty with which the horizontal wind speed incident upon the anemometer is known. It is required that an **uncertainty analysis** is carried out in accordance with the ISO guide to the expression of uncertainty [3] comprising both type A and type B uncertainty. The procedure for estimating the uncertainty of calibration is described in detail in Appendix II. The magnitude of the net uncertainty shall be assessed statistically and shall take account of:

- flow speed measurement uncertainty (Pitot tubes, Transducers, Air density evaluation, etc.)
- frequency measurements
- Wind tunnel calibration including blockage effect
- Flow variability in the vicinity of the anemometer

The combined type A and B uncertainties shall not be greater than 0.1m/s at 10m/s wind speed.

9 OPTIONAL SENSITIVITY TESTS

The **effect of temperature** on the calibration of an anemometer type should be investigated by calibrating the instrument under normal laboratory conditions and under extreme conditions i.e. at approximately 0^{o} C accomplished by artificially cooling of the anemometer. Thus the validity of the original calibration at normal conditions can be checked. Bearing friction can be assessed by replacing the rotor with a ‘flywheel’ and monitoring the deceleration behaviour.

Sensitivity checks should also be carried out in order to investigate the influence of **the vertical angle of the flow** attacking the anemometer. This can be accomplished by placing the anemometer at different tilt angles within the wind tunnel. Comparison of the results obtained with these from normal calibration will lead to the quantification of the effect that the vertical angle of attack has on the calibration results.

10 REPORTING FORMAT

The relevant **documentation** shall provide information on the procedure followed and the facility used for calibrating the anemometers (test report on the calibration campaign) and on the individual anemometer calibration (anemometer calibration report).

The **test report** on the calibration campaign shall contain the following **information** as a minimum:

- description of the wind tunnel;
- sketch of the wind tunnel showing the exact positions of anemometer and Pitot tube(s) in the test section;
- flow quality measurements;
- blockage correction factor;
- instrumentation certificates;
- measurement procedure;
- data evaluation procedure;
- repeatability documentation of the anemometer calibration;
- uncertainty analysis;
- deviations from this requirements.

The **calibration report** of an anemometer shall as a minimum contain the following **information:**

- make, type and serial number of the tested anemometer;
- tube diameter of the mounting system;
- make, type and serial number of external converters, if taken (i.e. frequency to voltage converters);
- name and address of the MEASNET institute;
- signatures from the persons who carried out the calibration, checked the results and approved their issue;
- name of the wind tunnel;
- name of the customer;
- environmental conditions during calibration (air temperature, air pressure and humidity);
- regression parameters (Offset, slope, regression coefficient, standard uncertainty in the slope and intercept and the covariance of the slope and intercept of the wind speed);
- tabular and graphical presentation (amplified) of all calibration points and regression results;
- uncertainty associated to each measuring point;
- reference to the corresponding calibration campaign report, date of the calibration;
- photo showing the anemometer and the mounting in the wind tunnel;

11 UNCERTAINTY CALCULATION

11.1 Definitions and Symbols

Definitions and symbols and grouped below in accordance with the sections in which they are introduced.

General Uncertainty

c_i	influence coefficient = $\partial f / \partial x_i$
q	sampled process
$r(x_i,x_j)$	correlation coefficient between uncertainty sources x_i and x_j
s	uncertainty of type A
$u(x_i,x_j)$	covariance between uncertainty sources x_i and x_j
u_c	combined uncertainty
x_i	measured uncertainty source
y	measurand, $y=f(x_i)$,

Anemometer Calibration Uncertainty

B	barometric pressure	[Pa]
C_h	head coefficient of the Pitot tube	
k_c	pressure correction factor relating the Pitot-tube position to the anemometer position,	
k_f	flow correction factor which *inter alia* may take account of - wind tunnel blockage (Maskell's factor) - horizontal wind shear - global corrections obtained by cross checking against a quality, reference tunnel	
K	transformation factor in measurement chain, reciprocal of gain	
p_k	dynamic pressure measured by a reference Pitot static tube	[Pa]
P_W	vapour pressure	[Pa]
R_A	gas constant (dry air)	287.1 [J/(kgK)]
R_W	gas constant (water vapour)	461.5 [J/kgK)]
$S_A(\bar{v})$	type A uncertainty in mean wind speed	[m/s]
T_k	is the prevailing temperature	[K]

u_f	standard uncertainty in tunnel correction factor k_f	
u_t	standard uncertainty in wind tunnel flow correction factor k_c	
u_h	standard uncertainty in Pitot static tube head coefficient C_h	
$u_{c,B}$	combined uncertainty of category B	
v	mean wind speed	[m/s]
$\bar{v}$	mean wind speed	[m/s]
v_k	value of wind speed for sample number k	[m/s]
$V_{x,k}$	digital bit representation of the k'th sample of variable x.	
x_k	value of the physical parameter corresponding to $V_{x,k}$	
x,t	subscript for K corresponding to the transducer transformation	
x,s	subscript for K corresponding to the signal conditioning transformation	
x,d	subscript for K corresponding to the digital data sampling transformation.	
$u_{p,t}$	standard uncertainty in conversion factor $K_{p,t}$	
$u_{p,s}$	standard uncertainty in conversion factor $K_{p,s}$ (similarly for other $K_{subscript}$ factors)	
ρ	air density	[kg/m^3]
φ	relative humidity	[-]

Regression Analysis

A	slope constant chosen to minimise the mean square difference between samples of y and the fitted function $f(x)$
B	offset constant chosen to minimise the mean square difference between samples of y and the fitted function $f(x)$
$COV(\ )$	Covariance between two quantities
r	correlation coefficient
S_N,	precision index or standard error (otherwise uncertainty in mean value)
x_a	subsequent 'field' measurement of variable x
x_i	measured sample of independent variable, x
y_a	value of quantity y inferred from x_a and the best fit regression line
y_i	measured sample of dependent variable, y
$\bar{y}$	mean value of N samples of dependent variable, y
σ_A^2	Variance in A, a measure of its uncertainty
σ_B^2	Variance in B, a measure of its uncertainty
σ_y^2	variance of y
$\sigma(y_a)$	standard uncertainty in the estimation of y_a

$\sigma_{\bar{y}}^{2}$ mean square deviation between sampled values of y and best fit function

11.2 Overview

An anemometer calibration is not particularly useful, if the calibration details are not supplemented with an estimation of their uncertainty.

This document prescribes a methodology for carrying out an assessment of this uncertainty. It is based on the principles outlined in the *ISO Guide to the Expression of Uncertainty, 1993 (First Edition) ISBN 92-67-10188-9* [Ref. 3]. The annex is split into four sections. The first explains basic principles and background theory. The second interprets this in the context of anemometer calibration. The third expands upon so called error type A. The fourth and final section shows by way of example how the overall theory can be applied in practice.

The contents of this supplement should be considered as forming an integral part of the calibration methodology outlined in the main MEASNET anemometer calibration document.

11.3 General Uncertainty Theory According to ISO Guide

In measurement, there are two types of uncertainty, type A which can be deduced objectively from the measurements themselves (this being related to statistical scatter), and type B which cannot.

Uncertainty may be described as the standard deviation of the probability distribution of the measurand, and is termed standard uncertainty.

Apart from very simple cases, it is rarely possible to measure a quantity (termed the measurand) directly, and values must be inferred from measurements of other parameters. The uncertainty in the measurand is the combination of the uncertainties which arise from the other quantities. In general, the combined uncertainty u_c of a measurand, y, which depends upon a variety of other parameters x_i such that $y=f(x_i)$, $i=1....N$, can be expressed as

$$u_c^2(y) = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u\left(x_i, x_j\right) \qquad (1)$$

$u(x_i,x_j)$ describes the elements in the covariance matrix,. $\partial f/\partial x_i$ is termed an influence coefficient, otherwise written equivalently as c_i, and defines how sensitive the measurand is to changes in the value of the measured parameter x_i.

It may be more useful in practice to work with correlation coefficients, $r(x_i,x_j)$, and this allows the expression to be represented in terms of pure variances as:

$$u_c^2 = \sum_{i=1}^{N} c_i^2 u^2\left(x_i\right) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j u\left(x_i\right) u\left(x_j\right).r\left(x_i, x_j\right) \qquad (2)$$

Very often correlation coefficients can be assumed to be equal either to zero or to unity. If different error sources are totally uncorrelated (ie $r(x_i,x_j)=0$, meaning that changes in one will not give rise to a change in another), then the expression reduces to:

$$u_c^2 = \sum_{i=1}^{N} c_i^2 u^2(x_i) \tag{3}$$

In the unusual case of all the error sources being fully correlated (ie $r(x_i,x_j)=1$) then the combined uncertainty reduces to:

$$u_c^2 = \left(\sum_{i=1}^{N} c_i u(x_i)\right)^2 \tag{4}$$

Type A uncertainty is normally given the symbol *s*, and relates to the uncertainty attached to the mean value of a sampled process, *q*. It can be evaluated as:

$$s^2(\bar{q}) = \frac{1}{n}\frac{1}{n-1}\sum_{k=1}^{n}\left(q_k - \bar{q}\right)^2 \tag{5}$$

where *n* is the number of samples.

11.4 Uncertainty Theory Applied to Anemometer Calibration

When carrying out an anemometer calibration, the measurand is the mean wind speed 'seen by' the anemometer during definition of a calibration point. This value cannot be measured directly, and the main document describes a recommended methodology for derivation of the measurand's value.

Typically, the mean wind speed is related to measurements of dynamic pressure, temperature and barometric pressure and is evaluated as:

$$\bar{v} = k_f \frac{1}{n}\sum_{k=1}^{n} v_k = k_f \frac{1}{n}\sum_{k=1}^{n}\left(\frac{2k_c p_k R T_k}{C_h B_k k_\rho}\right)^{\frac{1}{2}} \tag{6}$$

where:

v_k	is the value of wind speed for sample number *k* [m/s]
k_f	is a flow correction factor which *inter alia* may take account of - wind tunnel blockage (Maskell's factor) - horizontal wind shear - global corrections obtained by cross checking against a quality, reference tunnel
p_k	is the pressure measured by a reference Pitot static tube [Pa]
k_c	is a correction factor relating the Pitot-tube position to the anemometer position, this being derived by wind tunnel calibration tests
R	is the gas constant for dry air 287.1 [J/KgK]
T_k	is the temperature [K] for sample k
C_h	is the head coefficient of the Pitot tube
B_k	is the barometric pressure. [Pa] for sample k

k_ρ is a factor which corrects the density term (B_k/RT_k) for humidity effects

The factor, k_ρ, which corrects the air density for humidity effects is given by

$$k_\rho = \left(1+\frac{\varphi_k P_w}{B_k}\left(\frac{R_A}{R_w}-1\right)\right) \approx \left(1-0.378\left(\frac{\varphi_k P_w}{B_k}\right)\right) \qquad (7)$$

where:

φ_k is the value of relative humidity for sample k[-]
P_w is the vapour pressure for the prevailing temperature [Pa]
R_w is the gas constant for water vapour (=461.50) [J/kgK]

P_w can be approximated by the empirical polynomial equation

$$Pw = 0{,}0000205 * \mathrm{EXP}(0{,}0631846 * T) \qquad (8)$$

where t is in [°K]

During a calibration, k_ρ is unlikely to change appreciably and can be evaluated using the mean values of B, φ, and T.

input	physical parameter	x_k
conversion	transducer, with quoted sensitivity	gain $1/K_{x,t}$
link	low level voltage	$x_k/ K_{x,t}$
conversion	signal conditioning with applied gain	gain $1/K_{x,s}$
link	high level voltage	$x_k/ K_{x,t}\, K_{x,s}$
conversion	data sampling with discrete quantisation	gain $1/K_{x,d}$
output	bit representation	$V_{x,k}= x_k/ K_{x,t}\, K_{x,s}\, K_{x,d}$

Table 11.1: *Typical measurement chain*

In practice, the measured quantities will be measured indirectly where the chain will consist of the elements shown in the Table 11.1.

Each conversion has an associated factor, so that:

$$x_k = (K_{x,t} K_{x,s} K_{x,d})V_{x,k} \qquad (9)$$

where

x_k is the value of the physical parameter corresponding to input
$V_{x,k}$ the bit representation of the k'th sample of x.
K is a transformation factor
x,t is the subscript corresponding to the transducer transformation
x,s is the subscript corresponding to the signal conditioning transformation
x,d is the subscript corresponding to the digital data sampling transformation.

Taking these conversion expressions and inserting them into the physical parameter relationship gives:

$$\bar{v} = k_f \frac{1}{n}\sum_{k=1}^{n}\left(\frac{2k_c\,(K_{p,t}K_{p,s}K_{p,d}V_{p,k})\,R\,(K_{T,t}K_{T,s}K_{T,d}V_{T,k})}{C_h\,(K_{B,t}K_{B,s}K_{B,d}V_{B,k})\,k_\rho}\right)^{\frac{1}{2}} \qquad (10)$$

or

$$\bar{v} = k_f \left(\frac{2k_c\,(K_{p,t}K_{p,s}K_{p,d})\,R\,(K_{T,t}K_{T,s}K_{T,d})}{C_h\,(K_{B,t}K_{B,s}K_{B,d})\,k_\rho}\right)^{\frac{1}{2}} \frac{1}{n}\sum_{k=1}^{n}\left(\frac{V_{p,k}V_{T,k}}{V_{B,k}}\right)^{\frac{1}{2}} \qquad (11)$$

Each of the coefficients on the left hand side of the summation sign has associated uncertainty. The problem reduces to one of estimating the value of the contributory uncertainties, and deciding on how to amalgamate them, taking into account their degree of correlation. All these uncertainties will be of type B.

The right hand side of the expression relates to the derivation of a statistical mean, and the mean value will have associated type A uncertainty. This uncertainty, s_A is derived as follows:

$$s_A\left(\bar{v}\right) = k_f \left(\frac{2k_c\,(K_{p,t}K_{p,s}K_{p,d})\,R\,(K_{T,t}K_{T,s}K_{T,d})}{C_h\,(K_{B,t}K_{B,s}K_{B,d})\,k_\rho}\right)^{\frac{1}{2}} s\left(\bar{V}\right) \qquad (12)$$

where

$$s^2\left(\bar{V}\right) = \frac{1}{n}\frac{1}{n-1}\sum_{k=1}^{n}\left(\left(\frac{V_{p,k}V_{T,k}}{V_{B,k}}\right)^{\frac{1}{2}} - \bar{V}\right)^2 \qquad (13)$$

and

$$\bar{V} = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{V_{p,k}V_{T,k}}{V_{B,k}}\right)^{\frac{1}{2}} \qquad (14)$$

Returning to type B errors and referring to the expanded expression for mean wind speed, the sensitivity factors for the numerator coefficients are easily derived by partial differentiation and these equate to half the derived mean wind speed divided by the coefficient value, eg:

$$\frac{\partial\bar{v}}{\partial k_c} = \frac{1}{2}\frac{\bar{v}}{k_c} \qquad (15)$$

The only exception is for k_f which not being within the square root does not have the half factor:

$$\frac{\partial \bar{v}}{\partial k_f} = \frac{\bar{v}}{k_f} \tag{16}$$

The sensitivity factors for the denominator coefficients are similar, but have a negative sign, eg

$$\frac{\partial \bar{v}}{\partial C_h} = -\frac{1}{2}\frac{\bar{v}}{C_h} \tag{17}$$

Most of the uncertainty sources of type B can be regarded as being fully independent (*r=0*). One exception is the digital data system conversion uncertainty which, leaving aside the quantisation component, is likely to be fully correlated across all three data channels. Another exception is the humidity correction factor k_ρ which has a functional dependency on temperature and barometric pressure. Treating them as fully uncorrelated however, is conservative and will typically introduce very slight overall overestimation of uncertainty.

Defining, the type B uncertainty sources as follows:

$u_{p,t}$ standard uncertainty in conversion factor $K_{p,t}$
$u_{p,s}$ standard uncertainty in conversion factor $K_{p,s}$

.......and similarly for all the other $K_{subscript}$ conversion factors. Additionally, defining:

u_f standard uncertainty in tunnel correction factor k_f
u_t standard uncertainty in wind tunnel calibration coefficient k_c
u_h standard uncertainty in Pitot tube head coefficient C_h
u_ρ standard uncertainty in humidity correction to density, k_ρ

allows the combined uncertainty, $u_{c,B}$ of the category B errors to be expressed as:

$$\begin{aligned} u_{c,B}^2\left(\bar{v}\right) &= c_f^2 u_f^2 + c_t^2 u_t^2 + c_{p,t}^2 u_{p,t}^2 + c_{p,s}^2 u_{p,s}^2 + c_{p,d}^2 u_{p,d}^2 + c_{T,t}^2 u_{T,t}^2 \\ &+ c_{T,s}^2 u_{T,s}^2 + c_{T,d}^2 u_{T,d}^2 + c_h^2 u_h^2 + c_{B,t}^2 u_{B,t}^2 + c_{B,s}^2 u_{B,s}^2 + c_{B,d}^2 u_{B,d}^2 + c_\rho^2 u_\rho^2 \end{aligned} \tag{18}$$

where the sensitivity factors, *c*, have subscripts which follow the same pattern as those for the uncertainty sources to which they refer.

Thereafter, the total combined uncertainty can be evaluated as:

$$u_c^2\left(\bar{v}\right) = u_{c,B}^2\left(\bar{v}\right) + s_A^2\left(\bar{v}\right) \tag{19}$$

11.5 Basics of Regression Analysis

Sections 3 and 4 made reference to type A and type B errors, mentioning that type A could be assessed objectively from the measurements themselves. At various points in the calibration and interpretation process, it may be relevant to consider type A error, particularly when there is scatter in any calibration data.

This section shows by way of reference to a general calibration how type A error can be assessed and interpreted.

Assume that during a calibration procedure N sample pairs, x_i, y_i, are measured where x is the independent and y the dependent quantity. It is assumed inherently that x can be measured to high accuracy whereas y can have appreciable uncertainty. The mean value of the sample of y is given by

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{20}$$

which does not necessarily coincide with the true mean of the process, Y.

The spread of the distribution of y is described by its variance given by

$$\sigma_y{}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2 \tag{21}$$

which is equivalent to the square of the standard deviation.

For a Gaussian distribution 68.3 per cent of samples will lie within one standard deviation of the mean, 99 per cent within 2.58 standard deviations and 99.7 per cent within three standard deviations.

Closely related to the variance is the standard error, S_N, or precision index, given by

$$S_{N^2}(\bar{y}) = \frac{1}{N}\frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2 \tag{22}$$

which provides a measure of the accuracy with which the mean value is known. Here a 68.3 per cent confidence level is attached to the sample mean lying within one standard error of the true mean. This holds true for all distributions, not only for Gaussian processes.

Turning to regression analysis, for convenience, let $V_{x,y}$, be defined as:

$$V_{x,y} = \frac{1}{N^2}\left(N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i\right) \tag{23}$$

The purpose of linear regression analysis is to fit the sampled points x_i, y_i, $I = 1 \ldots N$ by the function

$$y(x) = Ax + B \tag{24}$$

where A and B are constants which are chosen to minimise the mean square difference between the values y_i and $y(x_i)$. It can readily be shown that the value of A that gives the best fit is given by

$$A = \frac{V_{x,y}}{V_{x,x}} \tag{25}$$

and that the corresponding best choice for *B* is

$$B = \bar{y} - A\bar{x} = \frac{V_{x,xy} - V_{xx,y}}{V_{x,x}} \tag{26}$$

Of course, the chosen *A* and *B* values are only best estimates and as such have associated uncertainly. The variance in *A* is given by

$$\sigma_A{}^2 = \frac{\sigma_{\bar{y}}{}^2}{\mathrm{V}_{\mathrm{x,x}}} \tag{27}$$

whilst that in *B* is given by

$$\sigma_B{}^2 = \frac{\overline{x^2}\sigma_{\bar{y}}{}^2}{\mathrm{V}_{\mathrm{x,x}}} \tag{28}$$

Equation 7.24 indicates that A and B are not independent since they are related via the mean values of the dependent and independent quantities. Thus a covariance exists defined by

$$COV(A,B) = \frac{-\bar{x}\sigma_{\bar{y}}{}^2}{V_{x,x}} \tag{29}$$

The mean square deviation between the sampled independent values, y_i, and those expected from the best linear fit, $y(x_i)$, appears in the above expressions and can be evaluated as

$$\sigma_{\bar{y}}{}^2 = \frac{1}{N-2}\left\{V_{y,y} - \frac{(V_{x,y})^2}{V_{x,x}}\right\} \tag{30}$$

A useful indication of the linearity of the relationship connecting the calibration points x_i, y_i is given by the correlation coefficient, *r*, given by

$$r = \frac{V_{x,y}}{\sqrt{V_{x,x}V_{y,y}}} \tag{31}$$

For $r = 1$ or $r = -1$ the points are perfectly described by a linear relationship, whilst for $r = 0$ the quantities are uncorrelated, although they may well be connected via a non-linear relationship.

Commonly, having established the regression coefficients, *A* and *B*, we are interested in using them to infer the value of the dependent quantity when only the independent quantity is known. For instance, assume that measurements x_a and x_b are made from which values y_a and y_b are derived using equation (24). It can readily be shown that there will be a covariance between the two estimates,

$$COV(y_a,y_b)=x_a x_b \sigma_A^2 + (x_a + x_b)COV(A,B) + \sigma_B^2 \tag{32}$$

which indicates that uncertainties in the two estimates are linked. is given by its standard deviation, defined by

$$\sigma(y_a) = \{x_a^2 \sigma_A^2 + \sigma_B^2 + 2x_a COV(A,B)\}^{½} \tag{33}$$

This is the major result of interest, since it indicates the degree of confidence attached to the value of a quantity which has been estimated from the accurate measurement of another variable.

11.6 Example

Ideally, the above theory should be applied independently to each wind speed calibration condition used in a calibration test. For this example, take a notional calibration point of 10 m/s using a wind tunnel rated at 25 m/s.

Table 11.2 below deals with each uncertainty source in turn, dealing first with those of type B.

To avoid repetition, a detailed assessment of barometric pressure measurement has been left out, as it can be dealt with in the same way as temperature measurement.

Error Source, u_i	Discussion	Value, u_i	Sensitivity Value, c_i	$u_i c_i$ (m/s)
u_f, wind tunnel correction factor, k_f	A comparison with a good tunnel (e.g. the NLR facility) might show a correction factor of 0.5% on wind speed is needed, i.e. k_f=1.005. It is suggested that a standard uncertainty of half the difference between the corrected and uncorrected value should be applied.	0.0025	$c_f=v/k_f$ =10m/s/1.005 =9.95m/s	0.025
u_t, wind tunnel calibration factor, k_c	Wind tunnel calibration can be carried out by using two Pitot tubes, one at the permanent reference position and one at the location to be occupied by the test anemometer. By swapping the two Pitot systems, all type B errors can be eliminated, and standard regression analysis can be applied to yield a correction factor (the intercept being forced through the origin) and a related type A standard uncertainty. Assume the correction has a value of 1.02 and the standard uncertainty is 0.01	0.01	$c_t=0.5v/k_c$ = 0.5*10/1.02 = 4.90 m/s	0.049

Error Source, u_i	Discussion	Value, u_i	Sensitivity Value, c_i	$u_i c_i$ (m/s)
$u_{p,t}$ pressure transducer sensitivity, $K_{p,t}$	Assume the pressure transducer is rated at 500N/m^2. At 10m/s wind speed, the pressure will be about 60N/m^2. Assuming the 'limits' on error are quoted by the manufacturer to be 0.2% of full scale (1N/m^2), and assuming this to relate to a triangular uncertainty distribution, then the equivalent standard deviation can be derived as 1*1/√6 or 0.40N/m^2. Assuming also that the transducer sensitivity, $K_{p,t}$ is 5000N/m^2 per V (100mV max output), then the standard uncertainty at 60N/m^2 $u_{p,t}$ equates to 33N/m^2 per V.	33	$c_{p,t} = 0.5\ v/K_{p,t}$ = 0.5*10/5000 =0.001	0.033
$u_{p,s}$ pressure transducer signal conditioning gain, $K_{p,s}$	Assume that the signal conditioning is designed to raise the maximum transducer output voltage (100mV) to the full scale range of the data system (10V), then the required gain is 100. Thus Kp,s =0.01. Assuming a standard uncertainty of 0.2%, this gives a value of $u_{p,s}$ of 0.00002	0.00002	$c_{p,s} = 0.5v/K_{p,s}$ =0.5*10/0.01 = 500	0.010
$u_{p,d}$ pressure transducer data sampling conversion $K_{p,d}$	The resolution of the data system is defined by the full scale values, e.g. 4096 bits for 10 volts or Kp,d of 0.00244 V per bit. The quantisation limits are half of this ie 0.00122 V per bit, and since a rectangular distribution is appropriate, the related standard uncertainty is 0.00122/√3 or 0.00704 V. For 10m/s wind speed, the voltage seen by the d/a system will be in the region of 1.2 V, giving a nominal bit value of 490. The conversion uncertainty $u_{p,d}$ is then no more than 0.000002 V/bit	0.000002	$c_{p,d} = 0.5v/K_{p,d}$ = 0.5*10/ 0.00244 = 2049	0.004
$u_{T,t}$ ambient temperature transducer, $K_{T,t}$	Temperature may appear to be somewhat difficult to handle, because whereas the foregoing theory assumed a zero offset in the relationship connecting temperature to transducer output, in reality a very high offset exists. Typically a temperature system might be quoted as giving a 4 to	n/a	$c_{T,t}=0.5v/K_{T,t}$ n/a	0.001

Error Source, u_i	Discussion	Value, u_i	Sensitivity Value, c_i	$u_i c_i$ (m/s)
	20 mA current range for a -20 to 30 ° C temperature range. Rather than trying to restructure the mathematics, it is possible to take a lateral approach. Assume the transducer is quoted as being good to 0.2 ° C. Assuming a triangular distribution, this relates to a standard uncertainty of 0.08 ° C. We know this is the temperature error attributable to the transducer, rather than the complete temperature chain. Going back to the basic equation for wind speed in terms of the physical T, B and p parameters, it is easy by varying T (from say 15 ° C, 288 ° K up to 15.08 ° C, 288.08 ° K) to determine the corresponding change in wind speed. This comes out, for 10m/s, as 0.001m/s. This value can be inserted directly in the last column of the table without reference to the third and fourth columns, which were based on the more general analytical approach.			
$u_{T,s}$ temperature signal conditioning gain, $K_{T,s}$	Assume the current output from the temperature sender unit is fed to a 500Ω precision resistor, to give a 2 to 10 volt output for the temperature range. The gain $K_{T,s}$ is thus 2mA/V. Assuming the resistor has a standard uncertainty of 0.2 ohm, then the gain will have a corresponding uncertainty of 0.0008mA/V.	0.0008	$c_{T,t}=0.5v/K_{T,s}$ =0.5*10/2 =2.5	0.002
$u_{T,d}$ temperature signal digital conversion, $K_{T,d}$	As for the pressure transducer signal line in the case above, the standard uncertainty of the quantisation is 0.00704V. For 15 ° C temperature, the voltage seen by the d/a system will be in the region of 7.6 V, giving a nominal bit value of 3113. The conversion uncertainty $u_{T,d}$ is then no more than 0.0000023 V/bit	0.0000023	$c_{T,d}=0.5v/K_{T,d}$ =0.5*10/ 0.00244 =2049	0.004
u_h Pitot tube head coefficient,	The head coefficient of a Pitot tube depends upon the angle of attack of the wind. Two error sources are possible, one related to the accuracy with which the	0.000997	$c_h=-0.5v/C_h$ =-.5*10/0.997	0.005

Error Source, u_i	Discussion	Value, u_i	Sensitivity Value, c_i	u_ic_i (m/s)
C_h	Pitot tube is set up in alignment with the mean flow direction, and the other due to turbulent variations in instantaneous flow direction. Assume the nominal head coefficient, C_h, is 0.997, and assume also that it is possible to deduce that the standard deviation on angle of attack is 2°. Relevant ISO standards suggest this will give rise to a 0.1% change in head coefficient.		=-5.015	
$u_{B,t}$ sensitivity of barometer, $K_{B,t}$	The barometer can be treated in much the same way as the temperature probe, since it to will have a large physical offset.		$c_{B,t}=-0.5v/k_{B,t}$	
$u_{B,s}$ signal condition-ing gain on barometer, $K_{B,s}$	similar approach as for other signal processing parameters		$c_{B,s}=0.5v/K_{B,s}$	
$u_{B,d}$ digital conversion of barometer signal, $K_{B,d}$	similar approach as for other data acquisition channels		$c_{B,d}=0.5v/K_{B,d}$	
s_A statistical uncertainty in the mean of the wind speed time series	Assume the turbulence intensity is 2%, and that 2Hz sampling over 30 seconds is used, giving 60 samples. The standard uncertainty in the mean value of 10m/s is then given by $\sqrt{(1/60)}*0.02*10$ or	0.026	1	0.026
u_ρ, humidity correction to density, k_ρ or u_φ, relative humidity, φ	It is possible to show that $c_\rho^2u_\rho^2$ is equivalent to $c_\varphi^2u_\varphi^2$ (where u_φ is the uncertainty in relative humidity and c_φ is the sensitivity of derived wind speed to humidity) if c_ρ is dominated by c_φ rather than c_B or c_T. This is normally the case. Assume relative humidity, φ, is measured from a hand-held meter as 50% to an accuracy of 5% within 95% confidence. $\varphi = 0.5$ and $u_\varphi = 0.025$	$u_\varphi = 0.025$	$c_\rho = 0.032$	0.001

Error Source, u_i	Discussion	Value, u_i	Sensitivity Value, c_i	$u_i c_i$ (m/s)
	$c_\varphi = \frac{\partial \bar{v}}{\partial k_\rho} . \frac{\partial k_\rho}{\partial \varphi} = \frac{1}{2} \frac{\bar{v}}{k_\rho} 0.378 \frac{P_w}{B}$ from equation (7) At 15°C, P_w = 1700 [Pa] from (8) and assuming B = 1013 mbar = 101300 [Pa] k_ρ is evaluated as 0.997 and c_φ (at 10m/s) is 0.032			

Table 11.2: Example of evaluation of anemometer calibration uncertainty

The combined uncertainty as shown in part 2 can be obtained by taking the root mean square of the contributory uncertainties in the right hand column. For the values which have been dealt with, this amounts to 0.07m/s.

The example shows that type B error is liable to dominate. Extending the calibration period can help reduce the type A uncertainty, but will have no effect on type B. Furthermore, type B error sources, although not correlated with one another for a particular wind speed, are fully self correlated across wind speeds, meaning that good apparent calibrations (good straight lines) can be obtained, whilst still retaining significant uncertainty.

The foregoing practical analysis can help identify where the major error sources are (in this case the wind tunnel calibration and the pressure transducer), and also whether the uncertainty sources are balanced.

12 EXAMPLE OF A CALIBRATION REPORT

<MEASNET INSTITUTE>
<NAME>
<MEASNET institute department>
<Address>
Tel:< >, fax: <>

Anemometer Calibration
Technical Report No. <*Report Num.*>

FOR: *<Customer Name>*
<Customer's Address>

IDENTIFICATION: **<Anemometer make and type>**
<Anemometer serial number>

DATE OF CALIBRATION: **24/07/1997**

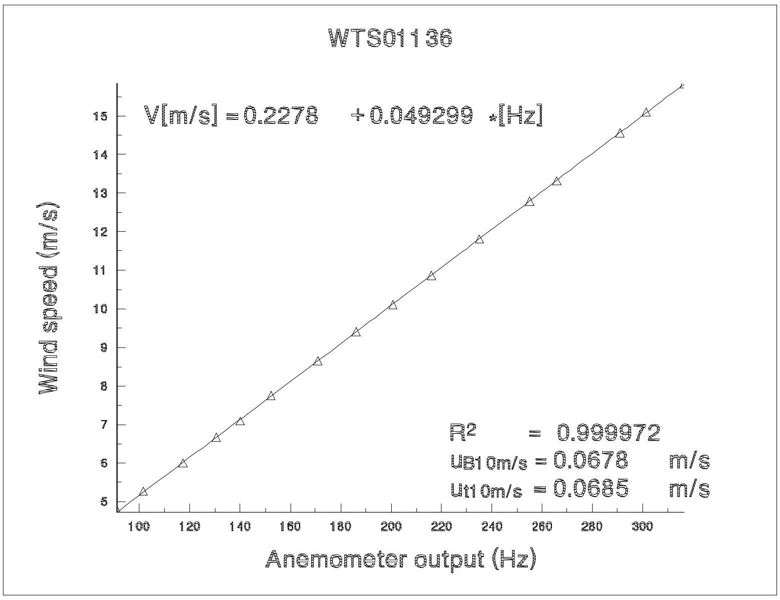
A. Measurements

Measurements were made according to the guidelines set by the MEASNET network at the Wind Tunnel of *<name of the MEASNET institute or the co-operating facility>*.
The reference velocity was measured using an *<reference instrument identification>*.
The anemometer was placed on the standard mounting pillar of the test section *<shape, dimensions>*. Before calibration, the anemometer was run at a steady wind tunnel velocity of 10 m/sec for 15 minutes in order to avoid the effect that large temperature variations may have on the mechanical friction of the anemometer bearings. Calibration was performed under both rising and falling wind speed in the range of 4 to 16 m/sec. The sampling frequency was 1 Hz and the sampling interval 30 sec. Before collecting data at each wind speed, 1 minute delay was allowed for stable conditions to become established.
The calibration campaign is described at Report *<Report title>*

B. Results, Graphical Presentation

Anemometer type: <Anem. type> s.n.: <Anem s.n> Code: <Anem. code>
Test date: 7-24-1997 Time (start) : 11:07:55.23
Wind tunnel air temp. ($_o$C): 31.30 Rel. humidity (%): 37.03 Atm. pressure(hPa): 993.46

Regression coefficients
a ($m*sec^{-1}/Hz$) : 0.049299
a_0 ($m*sec^{-1}$): 0.2278
R^2 : 0.999972 St. err. in Y: 0.0199 m/s



Total Uncertainty in reference wind speed

U_{ref}(m/sec)	u_{tot} (m/sec)
4.744	0.0837
5.993	0.0713
7.091	0.0664
8.646	0.0653
10.104	0.0685
11.813	0.0742
13.321	0.0812
15.844	0.0932
15.101	0.0932
14.563	0.0954
12.792	0.0783
10.865	0.0706
9.408	0.0666
7.752	0.0654
6.667	0.0681
5.269	0.0775

Equipment used	Type	s.n	code
Pitot tube:	<Type>	<s.n>	<code>
Dif. pressure sensor:	<Type>	<s.n>	<code>
Temperature sen.	<type>	<s.n>	<code>
Barometer	<type>	<s.n>	<code>
A to D card	<type>	<s.n>	<code>
Counter card	<type>	<s.n>	<code>

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C. Results, Tabular Presentation and Uncertainty

Reference wind speed (m/s)	Uncertainty in reference wind speed (m/s)	Anemometer output (Hz)	Residuals
4.7445	**0.0837**	**91.2667**	**0.0174**
5.9934	**0.0713**	**117.3667**	**-0.0204**
7.0906	**0.0664**	**140.0667**	**-0.0423**
8.6463	**0.0653**	**170.8000**	**-0.0018**
10.1041	**0.0685**	**200.7333**	**-0.0197**
11.8130	**0.0742**	**235.0333**	**-0.0017**
13.3205	**0.0812**	**265.9333**	**-0.0175**
15.8438	**0.0932**	**316.4333**	**0.0162**
15.1013	**0.0932**	**301.3000**	**0.0198**
14.5633	**0.0954**	**290.8333**	**-0.0023**
12.7919	**0.0783**	**254.9667**	**-0.0055**
10.8652	**0.0706**	**215.9333**	**-0.0079**
9.4081	**0.0666**	**185.9333**	**0.0140**
7.7515	**0.0654**	**152.3000**	**0.0155**
6.6668	**0.0681**	**130.5667**	**0.0022**
5.2687	**0.0775**	**101.5667**	**0.0338**

Uncertainty in reference and speed due to the uncertainty in the measurement of wind tunnel air relative humidity is not included to the values presented in this table.

D. Photo of the Anemometer in the Wind Tunnel

PHOTO OF THE ANEMOMETER

13 REFERENCES

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